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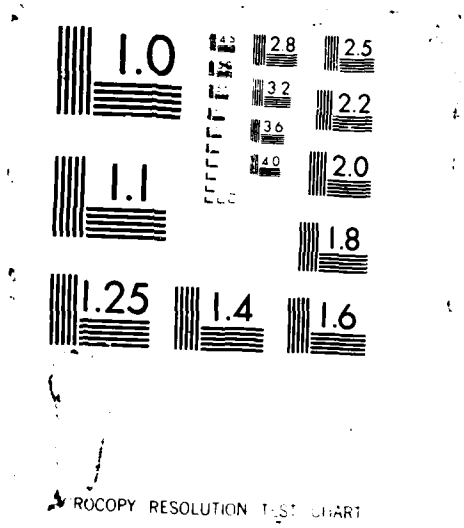
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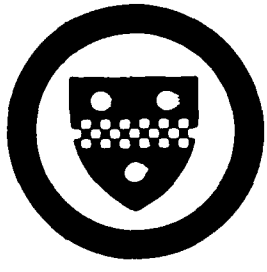

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Summary

The exact likelihood of the data coming from a moving average model of order is complicated. In this paper, we propose a method of estimation of the parameters of a moving average model of order one based on the approximate likelihood of the data and on the simulation of a pair of random variables. Some comparisons were made of this method with some well known methods for moderate sample sizes. A computer program is appended which is helpful in using this method.

Key words : Moving average model, Exact Likelihood, Approximate Likelihood, Simulation.

A NEW METHOD OF ESTIMATION IN A MOVING AVERAGE MODEL OF ORDER ONE

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1. Introduction

Estimation of parameters of a moving average model is a delicate subject fraught with complications. There are several methods available in the literature for estimating the parameters of a moving average model. Many of these methods require an enormous amount of computational effort. Though most of these methods provide consistent estimators of the parameters, it is of some interest to compare their performance for moderate sample sizes. McClave (1974) compared the performance of some five methods of estimation (Durbin (1959), Walker (1961), Hannan (1969), Parzen and Clevenson (1970), and Anderson (1973)) by simulating a first-order moving average model

$$Y_t = e_t + \beta e_{t-1}, \quad t = 0, \pm 1, \pm 2, \ldots ,$$

where e_t, $t = 0, \pm 1, \pm 2, \ldots$ is an independent identically distributed gaussian process with mean zero and variance $\sigma^2 = 1$, for $\beta = 0.5$ and 0.9, taking the sample size $N = 100$. McClave (1974) recommends the use of Parzen-Clevenson's, Hannan's and Walker's methods in that order for estimation. All these methods use approximations of one kind or another for the true expressions involved. As a result, these methods have inherently a certain

degree of complexity in their execution requiring a careful choice of trial approximations. Moreover, looking at the simulation studies of McClave (1974), one notices that β being systematically under-estimated and this being more so in the case $\beta = 0.9$.

In this paper, we propose a new method of estimation of the parameter β and σ^2 in a moving average model of order one, and examine its performance vis-a-vis with those of five methods considered by McClave (1974). This method is much simpler to use and is as good as any of these five methods. We apply this method to the time series data "IBM closing stock prices" and fit a moving average model of order one to these data. See Box and Jenkins (1976, p.526). We also give the computer program used in the estimation of β and σ^2.

In an article under preparation, we show how this new method can be used to make inferences in the context of signal processing when the input forms a moving average model of order one.

2. Estimation

The basic problem is to estimate β and σ^2 based on a single realization y_i of Y_i, $i = 1,2,\ldots,N$. The random vector $Y^T = (Y_1, Y_2, \ldots, Y_N)$ has a N-variate normal distribution with mean vector $\underset{\sim}{0}$ and dispersion matrix

$$\Sigma_N = \sigma^2 \begin{bmatrix} 1+\beta^2 & \beta & 0 & 0 & \ldots & 0 & 0 \\ \beta & 1+\beta^2 & \beta & 0 & \ldots & 0 & 0 \\ 0 & \beta & 1+\beta^2 & \beta & \ldots & 0 & 0 \\ & \ldots & & & \ldots & \ldots & \\ 0 & 0 & 0 & 0 & \ldots & 1+\beta^2 & \beta \\ 0 & 0 & 0 & 0 & \ldots & \beta & 1+\beta^2 \end{bmatrix}.$$

The likelihood of $y_1, y_2, \ldots, y_N$ involves Σ_N^{-1}. The inverse of Σ_N can be determined exactly but the exact likelihood of $y_1, y_2, \ldots, y_N$ is too unwieldy to handle. See Shaman (1969). In many of the methods currently used in estimating β of the moving average model of order one, Σ_N is replaced by a matrix whose inverse has a simple form. See Shaman (1969), McClave (1974), Godolphin (1978) and Godolphin and de Gooijer (1982). The replacement considered is given by

$$\Delta_N = \sigma^2 \begin{bmatrix} 1 & \beta & 0 & 0 & \ldots & 0 & 0 \\ \beta & 1+\beta^2 & \beta & 0 & \ldots & 0 & 0 \\ 0 & \beta & 1+\beta^2 & \beta & \ldots & 0 & 0 \\ & & \ldots & & \ldots & \ldots & \\ 0 & 0 & 0 & 0 & \ldots & 1+\beta^2 & \beta \\ 0 & 0 & 0 & 0 & \ldots & \beta & 1 \end{bmatrix}.$$

The inverse of Δ_N is given by

$$\Delta_N^{-1} = \frac{1}{\sigma^2(1-\beta^2)} \begin{bmatrix} 1 & -\beta & \beta^2 & -\beta^3 & \ldots & (-\beta)^{N-2} & (-\beta)^{N-1} \\ -\beta & 1 & -\beta & \beta^2 & \ldots & (-\beta)^{N-3} & (-\beta)^{N-2} \\ \beta^2 & -\beta & 1 & -\beta & \ldots & (-\beta)^{N-4} & (-\beta)^{N-3} \\ & & \ldots & & \ldots & \ldots & \\ (-\beta)^{N-1} & (-\beta)^{N-2} & (-\beta)^{N-3} & (-\beta)^{N-4} & \ldots & -\beta & 1 \end{bmatrix}$$

We call the resultant likelihood of $y_1, y_2, \ldots, y_N$ using Δ_N as "approximate likelihood".

Whittle (1953) has shown that the maximum likelihood estimator (using exact likelihood) of β is consistent, efficient and asymptotically

normal. (In all these deliberations, we assume that $|\beta| < 1$.) It can be shown that the estimator of β obtained using the approximate likelihood is consistent. See Niroomand Chapeh and Bhaskara Rao (1987). However, for moderate sample sizes the performance of the estimator based on approximate likelihood does not seem to be satisfactory for values of β close to zero and to one. We have simulated the moving average model of order one for $\beta = 0.1\ (0.1)\ 0.9$ and $\sigma^2 = 1$ taking $N = 100$, and then used the approximate likelihood to estimate β and σ^2. The symbols $\bar{\hat{\beta}}$ and $\bar{\hat{\sigma}}^2$ indicate averages of estimates of β and σ^2 ($=1$), respectively, taken over 100 repetitions of the above procedure. The symbols $\text{var}(\hat{\beta})$ and $\text{var}(\hat{\sigma}^2)$ indicate the variances of these 100 estimates of β and σ^2, respectively.

TABLE 1 : Mean and variances of the estimates using approximate likelihood

True Value of β	$\bar{\hat{\beta}}$	$\text{var}(\hat{\beta})$	$\bar{\hat{\sigma}}^2$	$\text{var}(\hat{\sigma}^2)$
0.1	0.086446	0.013927	0.972431	0.021727
0.2	0.175466	0.009216	1.012012	0.022227
0.3	0.308813	0.010092	0.974329	0.016091
0.4	0.403180	0.008703	0.979329	0.017651
0.5	0.502262	0.009645	0.994639	0.014661
0.6	0.604393	0.006919	0.980009	0.021174
0.7	0.695617	0.004870	0.984986	0.020208
0.8	0.777597	0.004416	1.008638	0.019917
0.9	0.857607	0.003533	1.076121	0.030506

Compare the above with the simulations performed by McClave (1974) for the cases $\beta = 0.5$ and 0.9. See also Godolphin and de Gooijer (1982).

The new method we want to propose has the following theoretical backing. Let $X^T = (X_1, X_2, \ldots, X_N)$ be a random vector normally distributed with mean vector $\underset{\sim}{0}$ and dispersion matrix Δ_N. Let $Z^T = (Z_1, Z_2, \ldots, Z_N)$ be a random vector normally distributed with mean vector $\underset{\sim}{0}$ and dispersion matrix given by

$$\Xi_N = \sigma^2 \begin{bmatrix} \beta^2 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ & \ldots & & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & \beta^2 \end{bmatrix}.$$

The random variables $Z_1, Z_2, \ldots, Z_N$ are independently normally distributed with $Z_2, Z_3, \ldots, Z_{N-1}$ being degenerate, all with common mean zero and the variance of each of Z_1 and Z_N being equal to $\beta^2\sigma^2$. Assume that X and Z are independent. It then follows that $Y \overset{d}{=} X + Z$, where $\overset{d}{=}$ indicates equality in distribution. If we have a realization z of Z, then we could obtain a realization x of X using y (given data) and z by taking $x = y - z$. Then we can work with the exact likelihood of x for estimating β and σ^2 instead of working with the approximate likelihood of y for the estimation of β and σ^2. One way of obtaining a realization of Z is by simulating Z once. The simulation of Z_1 and Z_N involves the unknown parameters β and σ^2. We can use some decent estimates of β and σ^2 to simulate Z_1 and Z_N. This is the basic idea behind the new method. Schematically, we outline the procedure as follows.

Step 1. Estimate β and σ^2 using the approximate likelihood of $y_1, y_2, \ldots, y_N$. Let $\hat{\beta}_{(1)}$ and $\hat{\sigma}^2_{(1)}$ be the respective estimates.

Step 2. Simulate Z_1 and Z_N each once, where Z_1 and Z_N are independently identically normally distributed with mean 0 and variance $\hat{\sigma}^2_{(1)}\hat{\beta}^2_{(1)}$. Let the realizations be denoted by z_1 and z_N.

Step 3. Define $x_i = y_i$ for $i = 2,3,\ldots,N-1$,

$= y_i - z_i$ for $i = 1,N$.

Step 1*. View $x^T = (x_1, x_2, \ldots, x_N)$ as a realization of $X^T = (X_1, X_2, \ldots, X_N)$. Estimate β and σ^2 using the likelihood of x. Let these estimates be denoted by $\hat{\beta}_{(2)}$ and $\hat{\sigma}^2_{(2)}$.

Keep repeating Steps 2 and 3 until two consecutive $\hat{\beta}$'s and $\hat{\sigma}^2$'s agree in a specified number of decimal places.

Some of the theoretical properties of these estimators are currently under investigation by the authors.

3. Simulation Studies

We simulated a moving average model of order one with $\beta = 0.1\ (0.1)\ 0.9$ and $\sigma^2 = 1$ taking $N = 100$, and used the above procedure to get estimates $\tilde{\beta}$ and $\tilde{\sigma}^2$ of β and σ^2, respectively. This is repeated 100 times. The following table gives the averages and variances of these estimates over 100 repetitions.

Table 2 : Mean and variances of the estimates obtained by using a combination of approximate likelihood and simulation of two dummy variables

True value of β	$\bar{\tilde{\beta}}$	$\text{var}(\tilde{\beta})$	$\bar{\tilde{\sigma}}^2$	$\text{var}(\tilde{\sigma}^2)$
0.1	0.0934	0.0139	0.9935	0.0227
0.2	0.1908	0.0084	0.9916	0.0191
0.3	0.3016	0.0076	1.0216	0.0226
0.4	0.3992	0.0098	1.0096	0.0130
0.5	0.5012	0.0085	0.9916	0.0183
0.6	0.6044	0.0081	1.0013	0.0201
0.7	0.6928	0.0072	1.0204	0.0271
0.8	0.7706	0.0042	1.0182	0.0203
0.9	0.8589	0.0034	1.0499	0.0240

The computer program for the above is available from the authors on request.

McClave (1974) recommends Parzen-Clevenson's method for estimating β with samples of moderate size. His simulation studies cover β =0.5 and 0.9 only. The following table gives his results (with the best in each category) along with the ones obtained above.

Table 3 : Mean and variances of estimates of β

Method	True value of β			
	0.5		0.9	
	Mean	Variance	Mean	Variance
Durbin's	0.4991	0.0104	0.8713	0.0076
Walker's	0.4920	0.0072	0.8414	0.0051
Hannan's	0.5025	0.0072	0.8540	0.0055
Parzen-Clevenson's	0.4912	0.0070	0.8404	0.0051
Anderson's	0.4997	0.0075	0.8324	0.0038
Approximate likelihood	0.5023	0.0096	0.8576	0.0035
Chapeh-Rao's	0.5012	0.0085	0.8589	0.0034

Note : In the first five methods, the averages were taken over 200 repetitions. In the last two methods, the averages were taken over 100 repetitions.

In conclusion, we remark that the execution of the new method is direct and simpler than the five methods examined by McClave (1974). There seems to be some improvement in the accuracy of the estimates obtained by using the new method than by using approximate likelihood especially in the lower reaches of β.

4. A Case Study

Box and Jenkins (1976, p.239 and p.526) fitted a moving average model of order one for the time series data "IBM Common Stock Closing Prices - Daily, 17th May 1961 to 2nd November 1962" after obtaining the first order differences of these series. The method they have used is the iterative least squares procedure and the model works out to be

$$\nabla Y_t = e_t + 0.09e_{t-1}$$

with residual variance equal to 52.2, where $\nabla Y_t = Y_t - Y_{t-1}$ and Y_t's are the original series. We have applied the method based on a combination of approximate likelihood and the simulation of two dummy variables to the first order differences of the above time series data and obtained

$$\hat{\beta} = 0.0867 \quad \text{and} \quad \hat{\sigma}^2 = 52.219.$$

In Section 5, we give the computer program we have used in this connection, and this program can be used to fit moving average models of order one by the new method. This program gives both the estimates of β and σ^2.

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McClave, J.T. (1974) - A comparison of moving average estimation procedures, Comm. Stat., 3, 865-883.

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5. COMPUTER PROGRAM

Language

FORTRAN 77

Description and purpose

Estimate the parameters of a moving average model of order one using a combination of approximate likelihood and simulation of two independent normal random variables.

Structure

SUBROUTINE GO5DDF(XM,SIG1) : gives a random sample from a normal distribution with mean XM and variance SIG1.

Formal parameters

XM	INTEGER	INPUT : ZERO
SIG1	REAL	INPUT : ONE

SUBROUTINE CO5NBF(FUN,2,X,F,TOL,W,20,IFAIL)

Formal parameters

X	Real array of dimension 2	Input : Initial guesses of parameters Output : gives estimates of the parameters
TOL	The accuracy	
W	The work space	
IFAIL	INTEGER	Output : a fault indicator equal to

IFAIL =	1	On entry, $N \leq 0$, or XTOL < 0.0, or LWA $\frac{1}{2}N(3N+$
IFAIL =	2	There have been atleast 200(N+1) evaluations.
IFAIL =	3	No further improvement in the solution is possi
IFAIL =	4	The iteration is not making good progress.

AUXILIARY ALGORITHM

CO5NBF uses the subroutine FUN(L,X.F,IFAIL) : it finds the zeros of two non-linear equations F(1) and F(2) in the unknown parameters θ and σ^2.

N = the total sample size.

```
C      Estimation of parameters of a moving average model of order one
       IMPLICIT  REAL*8      (A-H,O-Z)
       IMPLICIT  INTEGER*2   (I-N)
       DIMENSION XV(N=), S(N=), X(2), W(30), F(2), YV(N=), U(2), T(N=)
       COMMON S,N
       EXTERNAL FUN
       IFAIL = 0
       X1 = 0.0D0
       X2 = 0.0D0
       XM = 0.0D0
       TOL = DSQRT(X02AAF(0.0D0))
       open(5, file = 'data')
       OPEN(6, FILE = 'RESULTS')
       read(5,*) XV
       WRITE(6,201)
       N =
       DO  4  J = 1,N
4          S(J) = 0.0
       DO  5  I = 1,N
           L = N-I+1
       DO  5  K = 1,L
           J = K+I-1
5          S(I) = S(I)+XV(K)*XV(J)
       X(1) = 1D-1
       X(2) = 1.0D0
       CALL C05NBF(FUN,2,X,F,TOL,W,20,IFAIL)
       IF(ABS(X(1)-X1).LT.0.1D-2.AND.ABS(X(2)-X2).LT.0.1D-2) GO TO 90
       X1 = X(1)
       X2 = X(2)
100    SIG1 = X1*X1*X2
       DO  6  I = 1,2
C      Simulation of two independent random variables
       U(I) = G05DDF(XM,SIG1)
6      CONTINUE
```

```
      YV(1) = XV(1)-U(1)
      YV(N) = XV(N)-U(2)
      DO  7  I=2,N-1
      YV(I) = XV(I)
7     CONTINUE
      DO  9  J = 1,N
9         T(J)=0.0
      DO  10  I=1,N
          L=N-I+1
      DO  10  K=1,L
          J=K+I-1
10        T(I)=T(I)+YV(K)*YV(J)
      X(1)=1D-1
      X(2)=1.0D0
      CALL  CO5NBF(FUN,2,X,F,TOL,W,20,IFAIL)
      IF(ABS(X(1)-X1).LT.0.1D-2.AND.ABS(X(2)-X2).LT.0.1D-2)  GO TO 90
      X1=X(1)
      X2=X(2)
      GO TO 100
90    WRITE(6,202) X1,X2
201   FORMAT('1')
202   FORMAT('OFINAL RESULTS ARE :- BHAT', F10.6, 'SIG.SQ',F10.6)
      STOP
      END
      SUBROUTINE  FUN(L,X,F,IFAIL)
      IMPLICIT  REAL*8   (A-H,O-Z)
      IMPLICIT  INTEGER*2 (I-N)
      DIMENSION  F(2),S(N=),X(2)
      COMMON  S,N
      F(1) = -N*X(2)*(1.0D0-X(1)*X(1))+S(1)
      F(2) = X(1)*X(2)*(1.0D0-X(1)*X(1))-X(1)*S(1)
      DO  1  I=2,N
          F(1) = F(1)+2.0D0*(-1.0D0)**(I+1)*(X(1)**(I-1))*S(I)
          F(2) = F(2)+((-1.0D0)**I)*(X(1)**(I-2))*(I-1+(3-I)*X(1)*X(1))*S(I)
1     CONTINUE
      RETURN
      END
```

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The exact likelihood of the data coming from a moving average model of order is complicated. In this paper, we propose a method of estimation of the parameters of a moving average model of order one based on the approximate likelihood of the data on the simulation of a pair of random variables. Some comparisons were made of this method with some well known methods for moderate sample sizes. A comuter program is appended which is helpful in using this method.

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